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# POSITIONING THE SENSOR IN SAMPLING FLOWMETERS

Flow-rate measurements have very important meanings in industry. The sampling flowmeters are devices which can be used for measuring the flow-rate in situations where full-bore flowmeters cannot be used for technical or economical reasons. The main problem in the flow-rate measurement with the help of sampling flowmeters is optimal positioning of the sensor in the pipe. When we use a mathematical model of a sensor and a mathematical model of an object, we can theoretically optimize the position of the sensor in the pipe.

Keywords: flowmeters, mathematical models

## 1. INTRODUCTION

The flow-rate measurements have very important meanings in industry. The sampling flowmeters are devices which can be used for measuring the flow-rate in situations where full-bore flowmeters cannot be used for technical or economical reasons. For example, in a large diameter installation transporting drinking water an electromagnetic flowmeter is very expensive and sometimes emptying a pipe is not recommended. The sensor or sensors can be inserted into the pipe or clamped on it without stopping the flow [8]. The main purpose of mathematical modeling of flowmeter sensors [1] is the description of the flow phenomenon in various conditions.

The second purpose is to reproduce the measured value and estimate the total error for a concrete flowmeter. The sensor has a big influence on the mathematical model of the flowmeter, because in it the measured value is converted into a signal which can be detected by the measuring transducer. The sensors are called a primary device, and the measuring transducers are called secondary devices [2].

The main problem in the flow-rate measurement with the help of sampling flowmeters is optimal positioning of the sensor in the pipe. When we use a mathematical model of a sensor and a mathematical model of an object, we can theoretically optimize the position of the sensor in the pipe.

In this paper the position of the sensor in a closed pipe on the basis for local flow measurement is analyzed.

## 2. THE MATHEMATICAL MODEL OF VELOCITY DISTRIBUTION

For real flow measurement for turbulent flow more authors in different professional literature use the Prandtl formula:

$$v = v_m \left(1 - \frac{r}{R}\right)^{\frac{1}{n}}, \tag{1}$$

where: $r$ – current radius, $v_m$ – velocity along the pipe axis, $n$ – a number which depends on Reynolds number and roughness of the pipe wall, $R$ – pipe radius.

Miller [5] proposes that the value of $n$ for smooth pipes can be calculated as a function of the Reynolds number $n = f(Re)$:

$$n = 1,66 \log Re. \tag{2}$$

## 3. THE PRINCIPLE OF OPERATION OF SAMPLING FLOWMETER

The scheme of flow-rate calculation in a sampling flowmeter with one sensor is introduced in Fig. 1.

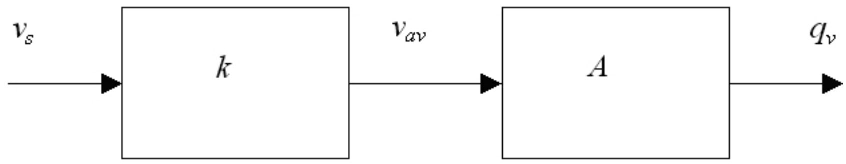


Fig. 1. The calculation of volume flow-rate in the sampling flowmeter: $v_s$ – local velocity measured with a sensor, $k$ – sensitivity factor, $v_{av}$ – average velocity in the pipe cross-section, $A$ – cross-section area of the stream in the pipe, $q_v$ – volume flow-rate.

The volume flow-rate in the sampling method of flow measurement is calculated from:

$$q_v = v_s k A, \tag{3}$$

where: $v_s$ – velocity value measured with the sensor, $k$ - sensitivity factor, $A$ - cross-section area of the stream in the pipe.

The sensitivity factor is defined by the formula:

$$k = \frac{v_{av}}{v_s}, \tag{4}$$

where: $v_{av}$ – average velocity in a cross-section of the stream in the pipe defined by:

$$v_{av}=\frac{1}{A}\iint_A v\ dA\ . \tag{5}$$

Local velocity measured with the sensor is defined by:

$$v_s=\frac{1}{A_s}\iint_{A_s} v\ \ dx\ \ dy\ , \tag{6}$$

where: $x$, $y$ – current coordinates.

For a greater number of sensors the volumetric flow-rate is calculated as the sum of partial flow-rates in individual areas which the whole cross-section was divided into.

## 4. METHODS OF POSITIONING THE SENSOR FOR MEASUREMENT OF FLOW RATE

For ideal fluid and stationary flow the velocity profile is constant throughout the cross-section of the pipe [8]. The velocity sensor can be placed at any point of the cross-section of the pipe and the measured velocity will be equal to average velocity. In real fluid the velocity profile is not uniform and the proper place for the velocity measurement is a frequent problem. Literature introduces two possibilities of positioning the sensor in the pipe [6]: the pipe centreline and a critical position. In [8] the author proposed three other possibilities of positioning the sensor in the pipe: equal area position, equal flow rate position, optimal position. The centreline position gives the maximum velocity. The velocity profile is flat and the error of positioning of the sensor has small influence on local velocity measurement. In [9] the relationship between average velocity and center velocity is introduced. For laminar flow it is 0.5 and is growing up rapidly for *Re* 2000 – 3000 (transversal flow) and for greater Reynolds number it is about 0.8. Critical position is a position in which the local velocity is the same as average velocity and the calculation of flow-rate is very simple. At this place the velocity profile is changing and each error in sensor positioning gives an error of local velocity measurement. The equal area position is used during the measurement of flow-rate with the help of the integrating method – where the whole cross-section is divided into many rings with the same cross-section area and local velocity is measured in each ring. The equal flow rate position is described in [8], and the optimal position is analyzed in [8].

## 5. REPRODUCTION OF THE MEASURAND FOR DIFFERENT POSITIONS OF THE SENSOR

### Centreline position

The centreline position means the natural position. The sensor is placed at the centre of the pipe and the centreline velocity is measured [6]. The sensitivity factor for centreline position for Prandtl formula is:

$$k_{cp}=\frac{2n^2}{(n+1)(2n+1)} \ . \tag{7}$$

When the Reynolds number increases for turbulent flow, the measured velocity approaches the average velocity [9]. The volume flow–rate is calculated from formula (3).

**Critical position**

The critical position means placing the velocity sensor at the point where local velocity is equal to average velocity [8]. For velocity distribution described by the Prandtl formula the average velocity can be calculated on the basis of formula (5) in the following way:

$$v_A=v_m\frac{2n^2}{(n+1)(2n+1)} \ . \tag{8}$$

The critical position is calculated from the formulae (1) and (8):

$$\frac{r_p}{R}=1-\left[\frac{2n^2}{(n+1)(2n+1)}\right]^n \ . \tag{9}$$

For this position the distance of the sensor is calculated from (3). The position of the sensor must change when the volume flow-rate changes. In a practical situation it means that we assume some volume flow-rate as nominal and for this value we estimate the sensor position, for which the value of sensitivity factor $k_{wp}=1$.

The measured velocity defined by Prandtl formula (1):

$$v_p=v_m\left(1-\frac{r_p}{R}\right)^{\frac{1}{n}} \ . \tag{10}$$

For nominal flow the "$n$" value is the nominal value $n_n$. In this situation the nominal position is:

$$\frac{r_{pn}}{R}=1-\left[\frac{2n_n^2}{(n_n+1)(2n_n+1)}\right]^{n_n} \ . \tag{11}$$

The measured velocity can be reproduced with comparison of the formulae (10) and (8):

$$v_{pn}=v_m\left[\frac{2n_n^2}{(n_n+1)(2n_n+1)}\right]^{\frac{n_n}{n}} \ . \tag{12}$$

The sensitivity factor can be calculated from formulae (12) and (8):

$$k_{wpm}=\frac{2n^2}{(n+1)(2n+1)}\left[\frac{(n_n+1)(2n_n+1)}{2n_n^2}\right]^{\frac{n_n}{n}} \ . \tag{13}$$

This factor depends on the actual value of "$n$", which can be calculated with an iterative method. The volume flow–rate is calculated from formula (3).

### Equal area position

The equal area position means placing the velocity sensor on the circle, which divides flow area into two equal parts [6]. The distance of this point from the circle centre is:

$$\frac{r_p}{R}=\frac{1}{\sqrt{2}}\ . \tag{14}$$

The velocity is measured at the distance from the pipe axis expressed by formula (14).

The sensitivity factor in the equal area position is:

$$k_{weap}=\frac{2n^2}{(n+1)(2n+1)\left(1-\frac{1}{\sqrt{2}}\right)^{\frac{1}{n}}}\ . \tag{15}$$

### Equal flow-rate position

The equal flow-rate position means placing the velocity sensor on the circle which divides the flow area into two parts, in which volume flow-rates are equal:

$$q_{vep}=\frac{q_v}{2}\ . \tag{16}$$

This method is recommended in [6].

Integrating (1) according to (5) and taking into account (16) we can calculate the equal flow position $r_q$:

$$\left[1-\frac{r_q}{R}\right]^{\frac{1+2n}{n}}+\frac{1+2n}{n}\frac{r_q}{R}\left[1-\frac{r_q}{R}\right]^{\frac{1+n}{n}}=\frac{1}{2}\ . \tag{17}$$

This position changes with Reynolds number and in Table 1 some values are introduced.

Table 1. The equal flow-rate position as a function of Reynolds number.

| *Re* | 10 000 | 100 000 | 1000 000 |
|---|---|---|---|
| $r_q/R$ | 0.666 | 0.674 | 0.679 |

The sensitive factor can be calculated from:

$$k_{wep}=\frac{2n^2}{(n+1)(2n+1)\left(1-\frac{r_q}{R}\right)^{\frac{1}{n}}}\ , \tag{18}$$

$k_{wep}$ – sensitivity factor in the equal flow-rate position.

## 6. CONCLUSIONS

In this article four ways of mounting a point velocity sensor are presented. This is such a position which enables to calculate the volume flow-rate on the basis of a mathematical model of velocity distribution and gives minimal measurement error. For nominal flow-rate the sensitivity factor is known, but for the same flow-rate the velocity distribution can change according to the changes of pipe wall roughness and viscosity of fluid, which depend on its temperature. In this situation we can estimate the scope of roughness changes and use the velocity distribution model in which these changes are taken into account. In the article there are four sensitivity factors calculated – for different positions of sensors. The centre line position I is the most natural. In other positions the distance of the sensor from the centre of the pipe depends on Reynolds number and the kind of mathematical model describing the velocity distribution. The introduced formulae enable to calculate flow-rate measurement errors for expected changes of Reynolds number for smooth pipes.

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